Simplifying Algebraic Expressions to Standard Polynomial Form:

How might we simplify an expression like

$$2(x-1)(x-2)(x+3) - (x^2-4)(x+2) + 5x^3 + (x^2-1)(x^2+1) - 10(x+1)$$

to standard polynomial form?

One approach* would be to break the problem into the following steps:

1. Multiply each term separately, keeping the parentheses:

FOIL   FOIL   FOIL   Multiply

$$2(x-1)(x-2)(x+3) - (x^2-4)(x+2) + 5x^3 + (x^2-1)(x^2+1) - 10x(x+1)$$

$$2(x^2-2x-x+2)(x+3)$$

$$(x^4+x^2-x^2-1)$$

$$(x^3+2x^2-4x-8)$$

$$(10x^2+10x)$$

2. Combine like terms:

$$2(x^2-3x+2)(x+3) - (x^3+2x^2-4x-8) + (x^4-1) - (10x^2+10x)$$

3. If necessary, repeat steps 1 and 2, until all terms have been multiplied:

Multiply each term by each term

$$2(x^2-3x+2)(x+3) - (x^3+2x^2-4x-8) \quad + \ (x^4-1) - (10x^2+10x)$$

$$2(x^3-3x^2+2x+3x^2-9x+6)$$

These terms   stay the   same

$$2(x^3-7x+6)$$

$$(2x^3-14x+12) - (x^3+2x^2-4x-8) + (x^4-1) - (10x^2+10x)$$

3. When all terms have been multiplied, distribute the negative ($-$) signs:

$$(2x^3-14x+12) + (-x^3-2x^2+4x+8) + (x^4-1) + (-10x^2-10x)$$

*If you already have an approach that works for you, that's great! Remember that there's almost always more than one right way to approach a math problem.

4. Now drop the parentheses:

$2x^3 - 14x + 12 - x^3 - 2x^2 + 4x + 8 + x^4 - 1 - 10x^2 - 10x$

↑

5. Find the term with the highest power of x. Find all terms with this power and combine.

Move the result to the beginning:

$x^4 + 2x^3 - 14x + 12 - x^3 - 2x^2 + 4x + 8 - 1 - 10x^2 - 10x$

↑   ↑

6. Repeat for the next highest power of x:

$x^4 + x^3 - 14x + 12 - 2x^2 + 4x + 8 - 1 - 10x^2 - 10x$

↑   ↑

7. Continue until all like terms have been combined:

$x^4 + x^3 - 12x^2 - 14x + 12 + 4x + 8 - 1 - 10x$

↑   ↑   ↑

$x^4 + x^3 - 12x^2 - 20x + 12 + 8 - 1$

↑ ↑ ↑

$x^4 + x^3 - 12x^2 - 20x + 19$

And we're done!

Exercises:

Simplify the following expressions:

1. $(x + 1)(x + 4) - (2x^3 - 1) + (x^2 - 1)(x - 2)$ 2. $6x^4 - 4(x - 1)(x^3 + 1) + 5(x^2 + 1) - x^3 - 10$

Answers: 1. $-x^3 - x^2 + 4x + 7$ 2. $2x^4 + 3x^3 + 5x^2 - 4x - 1$